MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2021/2022 SESSION

PPS0016 – INTRODUCTION TO PROBABILITY AND STATISTICS

(All sections / Groups)

12 AUGUST 2022
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of 5 pages (excluding the cover page) with 4 questions.
2. Answer all questions.
3. Unless stated otherwise, if an answer is given as a decimal, it should be rounded to four significant figures.
4. Write your answers in the Answer Booklet provided.
5. Show all relevant steps to obtain maximum marks.

Question 1 [20 marks]

Ambulance response time (ART) is defined as the period between emergency call received and ambulance arrival at scene. The following data give the ART (in minutes) in a city.

10	13	9	13	20	9	12	11	17	11
20	21	11	8	7	16	15	8	11	10
8	8	13	7	11	5	15	19	23	7
8	16	13	7	7	18	9	5	12	13
13	17	15	6	6	4	9	22	7	7

(a) Find the range of the data. [1 mark]

(b) By using 10 classes and the range in (a) above, determine the class size. [1 mark]

(c) Complete the following table with 10 classes. Use 4 minutes as the lower limit of the first class. [8 marks]

Class Limit	Class Boundary	Frequency	Cumulative frequency

(d) (i) Draw a histogram for the data. [8 marks]

(ii) Estimate the mode from your histogram. [2 marks]

Question 2 [20 marks]

(a) A study shows that 95% of car drivers wear seat belts. 64% of car drivers who involved in serious accidents die if not wearing a seat belt whereas 12% of those that do wear a seat belt die.

(i) Draw a tree diagram that shows the probability of events using the correct notation. S is the event that a driver wears a seat belt while D is the event that a driver dies. No need to show the leaves. [6 marks]

(ii) Find the probability that a driver dies in a serious accident but wears seat belts. [2 marks]

(iii) Find the probability that a driver survives a serious accident. [2 marks]

Continued...

(iv) Find the probability that a driver doesn't wear seat belts given he or she survives a serious accident. [2 marks]

(b) A random experiment has two events A and B such that $P(A \mid B) = \frac{2}{3}$, $P(B \mid A) = \frac{3}{8}$, and $P(A \cup B) = 1$.

(i) Find $P(A \cap B)$. [4 marks]

(ii) Draw a Venn diagram for the random experiment. Show the probability in each region. [4 marks]

Question 3 [20 marks]

(a) Two die are rolled. Let X denote the difference in the score between the two die.

(i) Construct the probability mass distribution of X in a table. [6 marks]

(ii) Find the probability that X is neither a prime number nor an odd number. [2 marks]

(iii) Find $E(X)$. [2 marks]

(b) A random variable X has probability density function given by:

$$f(x) = \begin{cases} \frac{1}{3}, & -1 \le x < 0 \\ \frac{1}{2}(x^2 + 1), & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the cumulative distribution function $F(x)$. [7 marks]

(ii) Use $F(x)$ to find $P(-0.5 < X < 0.5)$. [3 marks]

Question 4 [20 marks]

(a) On average, 180 cars per hour pass a specific point on a particular road in the morning rush hour. Due to roadworks at the city centre, if more than 5 cars pass the point in any one minute, congestion will occur at the city centre.

(i) Define the variable in the scenario. [1 mark]

(ii) What is the probability of congestion occurring? [4 marks]

Continued...

(iii) What should be the average cars per hour if the probability that no cars pass the point in any one minute is 0.1353? [3 marks]

(b) The diameter of an electric cable is normally distributed with mean 0.8 cm and variance 0.0004 cm^2. Use only the table provided in the appendix to find any related probability value.

(i) What is the probability that the diameter will exceed 0.81 cm? [3 marks]

(ii) The cable is considered defective if the diameter differs from the mean by more than 0.025 cm. What is the probability of obtaining a defective cable? [4 marks]

(c) A particular production process used to manufacture ferrite magnets is known to give 12% defective magnets on average. If 20 magnets are randomly selected,

(i) what is the probability that the number of defective magnets is not more than 3? [4 marks]

(ii) find the expectation of the number of defective magnets. [1 mark]

Question 5 [20 marks]

(a) The weights (in grams) of 5 apples in a packet are 150, 155, 155, 160, and 165.

(i) Construct the sampling distribution of the mean weight for samples of size 3 by filling in the following two tables. Use 155_1 and 155_2 to differentiate the two apples with weight 155. [7½ marks]

No	Sample	Sample mean
⋮		

Sample mean	Frequency	Probability of sample mean
⋮		

(ii) Show that the mean of the sampling distribution of the sample mean equals the mean of the population. [2 marks]

(iii) Find the standard deviation of the population. Hence, use it to find the standard deviation of the sampling distribution of the sample mean. [4 marks]

Continued...

(iv) For a sample size of 2, find the mean and standard deviation of its sampling distribution of sample mean. [2½ marks]

(b) (i) The combustion duration of a firework is normally distributed with a mean and standard deviation of 5.2 s and 0.4 s, respectively. For a sample size of 20, is the sampling distribution of sample mean of the combustion duration normal? Justify your answer. [2 marks]

(ii) The combustion duration of another firework has a skewed-left distribution with a mean and standard deviation of 5.2 s and 0.4 s, respectively. For a sample size of 20, is the sampling distribution of sample mean of the combustion duration normal? Justify your answer. [2 marks]

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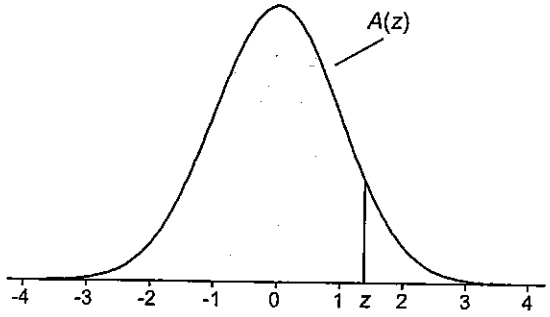
Appendix

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

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